AP Calc BC, Spring Break Practice Exam FRQ

Calculator Allowed

Question 1

Let f be the function given by $f(x) = 4x^2 - x^3$, and let
ℓ be the line $y = 18 - 3x$, where ℓ is tangent to the
graph of f. Let R be the region bounded by the graph of
f and the x-axis, and let S be the region bounded by the
graph of f, the line ℓ, and the x-axis, as shown above.
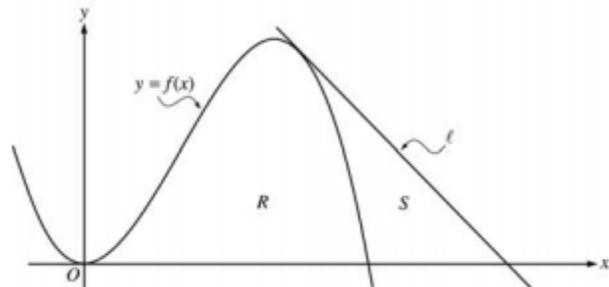
(a) Show that ℓ is tangent to the graph of $y = f(x)$ at
     the point $x = 3$.

(b) Find the area of S.

(c) Find the volume of the solid generated when R is revolved about the x-axis.

Question 2

The figure above shows the graphs of the circles $x^2 + y^2 = 2$ and
$(x-1)^2 + y^2 = 1$. The graphs intersect at the points $(1,1)$ and $(1,-1)$.
Let R be the shaded region in the first quadrant bounded by the two
circles and the x-axis.
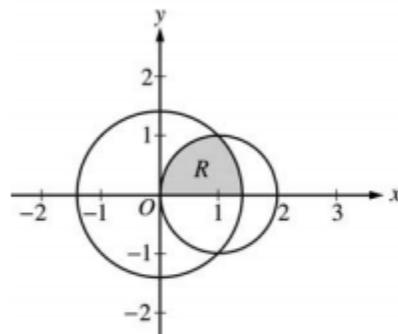
(a) Set up an expression involving one or more integrals with respect to
     x that represents the area of R.

(b) Set up an expression involving one or more integrals with respect to
     y that represents the area of R.

(c) The polar equations of the circles are $r = \sqrt{2}$ and $r = 2\cos\theta$, respectively. Set up an expression
     involving one or more integrals with respect to the polar angle θ that represents the area of R.

Question 3

A blood vessel is 360 millimeters (mm) long
with circular cross sections of varying diameter.
The table above gives the measurements of the
diameter of the blood vessel at selected points

Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

along the length of the blood vessel, where x represents the distance from one end of the blood vessel and
$B(x)$ is a twice-differentiable function that represents the diameter at that point.

(a) Write an integral expression in terms of $B(x)$ that represents the average radius, in mm, of the
     blood vessel between $x = 0$ and $x = 360$.

(b) Approximate the value of your answer from part (a) using the data from the table and a midpoint
     Riemann sum with three subintervals of equal length. Show the computations that lead to your answer.

(c) Using correct units, explain the meaning of $\pi \int_{125}^{275} \left(\frac{B(x)}{2} \right)^2 dx$ in terms of the blood vessel.

(d) Explain why there must be at least one value x, for $0 < x < 360$, such that $B''(x) = 0$.

Question 4

A particle moves in the xy-plane so that the position of the particle at any time t is given by

$$x(t) = 2e^{3t} + e^{-7t} \quad \text{and} \quad y(t) = 3e^{3t} - e^{-2t}.$$

(a) Find the velocity vector for the particle in terms of t, and find the speed of the particle at time $t = 0$.

(b) Find $\frac{dy}{dx}$ in terms of t, and find $\lim\limits_{t \to \infty} \frac{dy}{dx}$.

(c) Find each value t at which the line tangent to the path of the particle is horizontal, or explain why none exists.

(d) Find each value t at which the line tangent to the path of the particle is vertical, or explain why none exists.

Question 5

Let f be a function defined on the closed interval $[0,7]$. The graph of f, consisting of four line segments, is shown above. Let g be the function given by $g(x) = \int_2^x f(t)\,dt$.
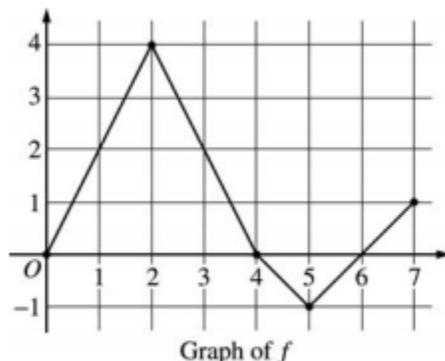
(a) Find $g(3)$, $g'(3)$, and $g''(3)$.

(b) Find the average rate of change of g on the interval $0 \le x \le 3$.

(c) For how many values c, where $0 < c < 3$, is $g'(c)$ equal to the average rate found in part (b)? Explain your reasoning.

(d) Find the x-coordinate of each point of inflection of the graph of g on the interval $0 < x < 7$. Justify your answer.

Graph of f

Question 6

The function f has a Taylor series about $x = 2$ that converges to $f(x)$ for all x in the interval of convergence. The nth derivative of f at $x = 2$ is given by $f^{(n)}(2) = \dfrac{(n+1)!}{3^n}$ for $n \ge 1$, and $f(2) = 1$.

(a) Write the first four terms and the general term of the Taylor series for f about $x = 2$.

(b) Find the radius of convergence for the Taylor series for f about $x = 2$. Show the work that leads to your answer.

(c) Let g be a function satisfying $g(2) = 3$ and $g'(x) = f(x)$ for all x. Write the first four terms and the general term of the Taylor series for g about $x = 2$.

(d) Does the Taylor series for g as defined in part (c) converge at $x = -2$? Give a reason for your answer.